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On parametrization and mixture laws for electron ionization coefficients

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Abstract. In this paper we discuss the methods of calculation of electron ionization coefficient data. First we give a summary of the data for analytic parametrization of ionization coefficients. Such data may be useful for particle and radiation detectors and for studies of breakdown voltages in gas dielectrics. We have applied extended Townsend formula to fit the experimental data for a number of gases and shown that it provides excellent fits for the entire range of E/N where data are available. We also tested the application of the common E/N and the common mean energy combination of data for pure gases to obtain ionization coefficients for mixtures. The standard combination procedure gives poor results in general but the common mean energy procedure provides an extended region of reasonable usefulness. Test calculations were made for Ar–CH_4 mixtures which were found to be the most difficult combination of the selected gases for the application of the mixture law.

PACS. 29.40.Cs Gas-filled counters: ionization chambers, proportional, and avalanche counters – 34.50.Fa Electronic excitation and ionization of atoms (including beam-foil excitation and ionization) – 34.50.Gb Electronic excitation and ionization of molecules; intermediate molecular states (including lifetimes, state mixing, etc.) – 51.50.+v Electrical properties (ionization, breakdown, electron and ion mobility, etc.)

1 Introduction

Over the past several years, there has been an increasing interest in developing techniques for reliable predictions of the electron ionization coefficients for the individual gases as well as for the mixtures of gases. The importance of such data originates from the development of a number of applications.

Ionization is certainly critical information for fluid plasma models that involve direct application of transport data [1–3]. In more complex models such as hybrid [4], Monte Carlo [5] and particle in cell [6], where cross-sections are employed, one needs comparisons with swarm data to normalize the cross-sections [7]. In particular, little care has been given to fitting of the ionization coefficients, which are the only data that can be used to normalize the total effect of excitation and dissociation cross-sections and which fix the high energy tail of the electron energy distribution function (EEDF) [8,9]. On the other hand, ionization is the critical rate coefficient required to model plasma maintenance.

Operation of gas discharges is usually established at the operating point where ionization can compensate losses and if there is attachment, the losses are easy to determine [10]. Appropriate selection of the reduced field E/N (where E is the electric field and N gas number density, given in units of Townsend (Td) — $1\ \text{Td} = 10^{-21}\ \text{Vm}^2$, $1\ \text{V/cmTorr} = 3.0341\ \text{Td}$ at 293 K) and mixture composition could lead to synergism i.e. breakdown potential greater than the breakdown potentials for pure constituent gases [10,11]. These studies were often based on approximate analytic approximations of the E/N dependence of ionization coefficient and would benefit greatly from improved analytic fits and mixture laws [10,11].

Another field where improved analytic representation of ionization coefficients and the corresponding mixture laws may be useful is in modelling the volt-ampere characteristics, and Paschen curves for low current discharges [12,13]. Such information is for example needed for extracting information on secondary electron yields [14–16] for conducting and for dielectric electrodes [17,18], for modelling cylindrical discharge geometries [19,20] and for verifying the applicability of the models in predicting properties of higher current discharges [21–23]. Most interesting recent application is in description of micro discharges for atmospheric non-equilibrium plasma processing and plasma displays operating in dc [24–26] and in rf [27] and for modelling of energy input into micro machines [28]. All these applications involve gas mixtures of different compositions.

Finally, the swarm data in pure gases and in mixtures has been used extensively in development of a wide range

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of particle and ionizing radiation detectors [29] including: ionization detectors [30], Geiger Müller counters [31], various types of proportional counters [32], gas phase electron multipliers [33], scintillation counters [34], spark and streamer chambers [35], wire and drift chambers [29,36] and different types of calorimeters including gas phase ionization calorimeters [37] and many variations involving gaseous electronics and gas discharges. The high resolution of multiwire detectors with the development of micro-strip gas chambers and micro-gap chambers [38] may allow a broad range of applications for medical imaging. While ionization coefficients are directly implemented in ionization chambers and electron multipliers, their applications may be extended for modelling and optimization of all particle and radiation detectors based on gas phase charge conduction.

In this paper, we discuss the methods that are currently used for calculation of ionization coefficients. In particular, we discuss the combination of the ionization coefficients for pure gases to provide data for a mixture.

2 Analytic representations of ionization coefficients for electrons in pure gases

Following the first studies of gaseous electronics, Townsend has proposed both the definition of the ionization coefficient (as a number of ionization events per unit distance per electron) and a possible analytic expression [39,40]. This expression is in the literature usually known as Townsend formula [41]. It may be written in a more modern form:

$$\frac{\alpha}{N} = A \exp\left[-\frac{B}{E/N}\right], \tag{1}$$

where A and B are constants characteristic for a particular gas. Originally, both electric field and the coefficients in the formula were normalized to the pressure (in Torr) (in early days of gaseous electronics E was sometimes denoted as X). However, in this paper we consistently normalize quantities to the gas number density as has been common in the previous forty years in physics of charged particle swarms [42,43].

In the first half of the twentieth century the formula (1) was commonly used to represent experimental data and also it was tested in numerous examples [44]. A and B may be obtained by fitting the experimental results with equation (1) and usually data from [41,45] were quoted in the literature even though those were sometimes based on older and less accurate experimental data. Often, however, the formula proved to be insufficient to fit the experimental data [40] and either different formulae were proposed or the ionization coefficient was fitted in segments [44].

In the second half of the century, in physics of swarms, tabulated experimental data were mainly used and the formula lost its importance. Nevertheless, its application continued in analytic models of low current discharges [13,14,21]. In cases of its failure numerous semi empirical forms were used. For example Phelps and coworkers [46] have used for hydrogen:

Table 1. Constants A and B from equation (1) for different gases; and range of E/N where the formula is valid. * (1 Td = 10^{-21} Vm^2).

gas	A [10^{-21} m^2]	B [Td]	range of E/N [Td]
He	8.5	234	100−750
Ne	13.3	337	300−1200
Ar	34.9	534	300−1800
Kr	47.3	667	300−3000
Xe	72.8	1000	600−2400
H_2	15.0	413	45−900
N_2	35.8	986	300−1800
O_2	19.7	576	150−400
CH_4	51.6	910	450−3000
CF_4	32.8	646	80−600

*Most of these data were obtained by Prof. Liebermann [54] who fitted our extended Townsend formula fits to experimental data (3) by a standard Townsend formula (1) in order to extend the range of validity of the data available in the literature [41,45].

$$\frac{\alpha}{N} = \frac{1.4 \times 10^{-20} \exp\left(-\frac{405}{E/N}\right)}{[(10^{-4}E/N)^{1.5} + 1]^{0.5}} \tag{2}$$

where α/N is in m^2 and E/N in Td, which was claimed to be valid below 1000 Td because of a possible non-equilibrium with the electric field for realistic experiments above that E/N.

As for modelling of the ionization coefficient or multiplication factor (gain = $\int_0^x \alpha(E(\rho)/N)d\rho$ — where ρ is a general spatial coordinate), nuclear physics community has offered a large number of different analytic forms since the need to employ analytic formulae was greater. These studies originate from first studies of proportional counters by Rose and Korff [47]. A large number of formulae has been developed. Few were based on different degrees of physical foundation (approximation) while most were purely semi-empirical. We will mention here the work of Zastawny [48,49], Kowalski [50], Mazed and coworkers [51,52] and Uozumi et al. [53] which have tried to apply their gain formula to gas mixtures. Charge gain may also be associated with the efficiency of scintillations [34] and gain formulae may be tested.

One may, however, conclude that Townsend formula is still the most popular as it has a clear and simple (albeit approximate) physical foundation, is simple to employ and allows analytic solution to the breakdown condition, describes ionization coefficient in a wide range of E/N and there is plenty of data for its parameters available in the literature [38,48,49].

2.1 Fitting parameters for ionization coefficients

Constants A and B, obtained by fitting the experimental results with equation (1) are presented in Table 1, along

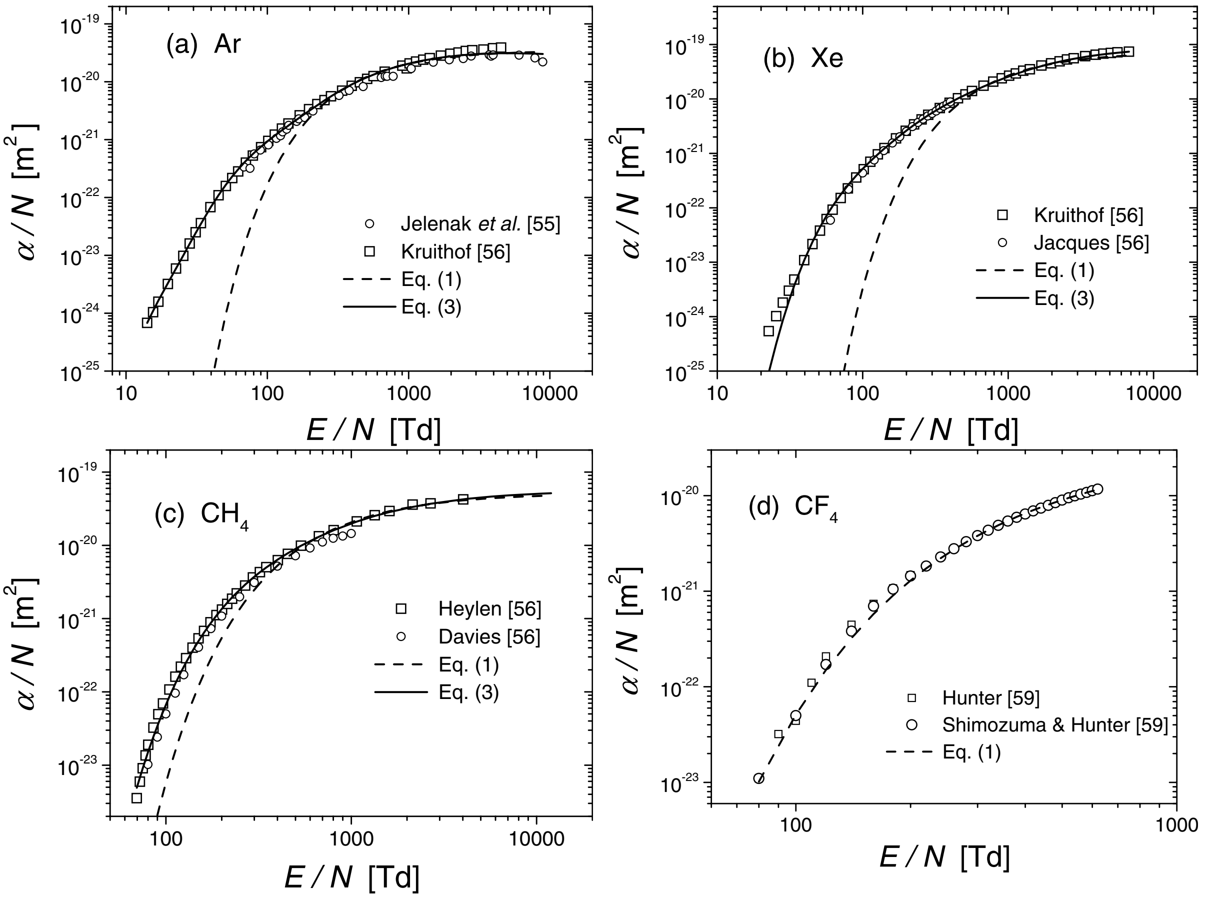

Fig. 1. Electron ionization coefficient α/N dependence on reduced electric field E/N for: (a) Ar; (b) Xe; (c) CH_4; (d) CF_4. The symbols show the experimental data, the dashed lines are obtained by fitting the experimental results with Townsend formula (1) and the solid lines represent fit by formula (3). Original references for data from compilations [56,59] may be found there and will not be cited here.

with the range of reduced electric field E/N where the experimental data for α/N are well reproduced [54].

It was suggested in the literature [48,49] that Townsend formula can describe accurately all the features of the ionization coefficient which led to its predominant use in modelling of particle and radiation detectors, gas discharges and collisional plasmas. However, if we observe fit by (1) shown in Figure 1, it is obvious that even the best fit cannot describe accurately the ionization in a wide range of reduced electric fields. Moreover, the decrease of ionization at high values of E/N [55], that is experimentally confirmed for a number of gases, cannot be reproduced by equation (1).

Phelps and Petrović [13,14] have used a semi-empirical formula that has several terms identical to Townsend's expression (1) but with quite different values of parameters, B_i in particular:

$$\frac{\alpha}{N} = \sum_i A_i \exp\left[-\frac{B_i}{E/N}\right]. \tag{3}$$

While one may seek the physical foundation in the existence of several electron groups, the B_i value gives an obvious E/N range where the term i is significant. At the same time A_i provides an insight of the maximum contribution of the term. Thus it is easy to estimate which approximate values of A_i and B_i may be required to fit the experimental data in a certain range of E/N. At the same time, one may expect that the dominant term would be similar to the standard Townsend formula fit for the same gas. Thus one may conclude that this formula (3) is more meaningful than choosing a complex analytic form that would have to be different for different gases and which would have coefficients that have no clear interpretation. It should also be noted that choosing a negative A_i would allow adjustments in both directions and even representation of decreasing α/N with E/N. In addition, the simple form (3) is easily extended to the formula for the electron gain (at least for uniform fields) which is usually not the case for other analytic forms proposed in the literature [50,52].

Constants A_i and B_i, obtained for different gases, are presented in Table 2. It is important to point out that the range of validity of equation (3) given in the table is limited mostly by the range of the experimental data available in the literature and not by the validity and flexibility of the formula itself.

Table 2. Constants A_i and B_i from equation (3) for different gases; and range of E/N where the formula is valid.

gas	$A_1[10^{-21}\,\mathrm{m}^2]$	B_1[Td]	$A_2[10^{-21}\,\mathrm{m}^2]$	B_2[Td]	$A_3[10^{-21}\,\mathrm{m}^2]$	B_3[Td]	$A_4[10^{-21}\,\mathrm{m}^2]$	B_4[Td]	range of E/N [Td]
He	1.7	75	8	350	/	/	/	/	10−900
Ne	0.6	45	4	150	10	560	−36	23000	10−9000
Ar	0.11	72	5.5	187	32	700	−15	10000	15−6000
Kr	0.8	115	13	300	43	1200	/	/	15−6000
Xe	6	250	30	700	57	2250	/	/	40−7000
H_2	15	413	/	/	/	/	/	/	45−900
N_2	20	800	26	1800	−70	10000	/	/	90−4000
O_2	9	490	9	630	100	10000	/	/	70−400
CH_4	27	600	30	1780	/	/	/	/	70−4000
CF_4	33	646	/	/	/	/	/	/	80−600

For the gases that we investigated, it was sufficient to use up to four exponential terms in equation (3). In the case of Ne, Ar and N_2 negative terms show up. Those describe the decrement of α/N at high E/N. We can also see that Townsend formula (i.e. only a single term in (3)) is satisfactory for description of ionization in H_2 and CF_4, though, the range of the available data in the literature that we selected for those gases was rather narrow.

Experimental data for ionization coefficients for several gases [56–59], along with best fits by equations (1) and (3) are shown in Figure 1. Evidently, the proposed extended Townsend fitting formula reproduces the experimental data better.

3 Predicting the ionization coefficients in gas mixtures

As mentioned in the introduction, most of the current applications that involve direct use of the ionization coefficients (i.e. convergence of hydrodynamic expansion is implied and the theory is local in respect to electric fields) involve mixtures of gases where gaseous electronics is used as the basis to engineer the required properties. Such examples may be found in development of gaseous dielectrics [60], in studies of gas breakdown [11], in modelling of glow and low current discharges, optimization of gas laser discharges, modelling of different types of discharges at atmospheric pressure [61] and of course in development of a broad range of particle and radiation detectors [53,62].

Application of most analytic formulae for electron gain calculation implies special fits for each of the mixtures [49]. Methods of calculation of ionization coefficients based on numerical solution of Boltzmann's equation and Monte Carlo simulations may not be practical in a number of applications. Thus, it is desirable to develop techniques to combine data for pure gases in order to obtain the data for a mixture.

The most widely used analytic method for predicting the ionization coefficients in mixtures of gases that was developed within the the gaseous dielectric community is the so-called Wieland approximation [10,63]:

$$\left[\frac{\alpha}{N}\left(\frac{E}{N}\right)\right]_{\mathrm{m}} = \sum_z x_z \left[\frac{\alpha}{N}\left(\frac{E}{N}\right)\right]_z, \qquad (4)$$

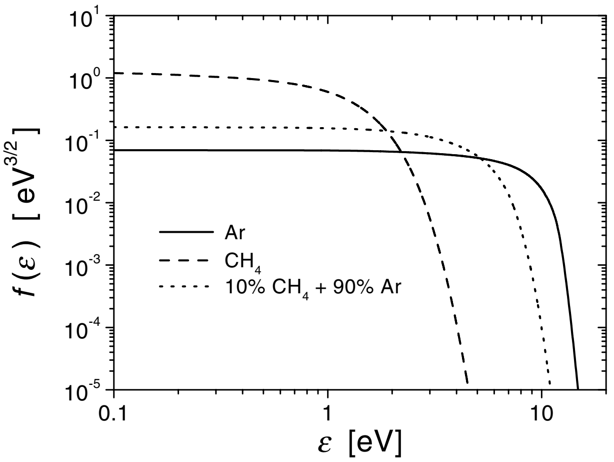

Fig. 2. Comparison of calculated electron energy distribution functions (EEDF) at $E/N = 10$ Td in Ar – solid line, CH_4 – dashed line and 10% CH_4 + 90% Ar mixture – dotted line. EEDF-s are calculated by using the ELENDIF Boltzmann code [71].

where x_z is the fraction of gas z in the mixture ($\sum x_z = 1$). The ionization coefficient for the mixture is simply calculated as a linear combination of ionization coefficients for the constituent gases weighted by the corresponding abundances.

In this respect, the formula appears to be analogous to the Blanc's law which is defined for drift velocities [64] though it may also be applied for diffusion coefficients by using Einstein's relations [65].

However, electron kinetics is very strongly affected by the electric field and also by inelastic collisions due to inefficiency of elastic collisions in the energy transfer. Therefore, for electrons it is likely that the distribution functions at the same E/N in two gases are quite different and the distribution function for the mixture is different from both (Fig. 2).

Consequently, mean electron energies for constituent gases and for the mixture may be considerably different (see Fig. 3). This is true in particular for moderate E/N (corresponding to mean energies between 0.2 eV and 5 eV) [66] while at higher E/N the condition is satisfied much better. As a result, Blanc's law has only been applied to ions as they are close to thermal equilibrium for

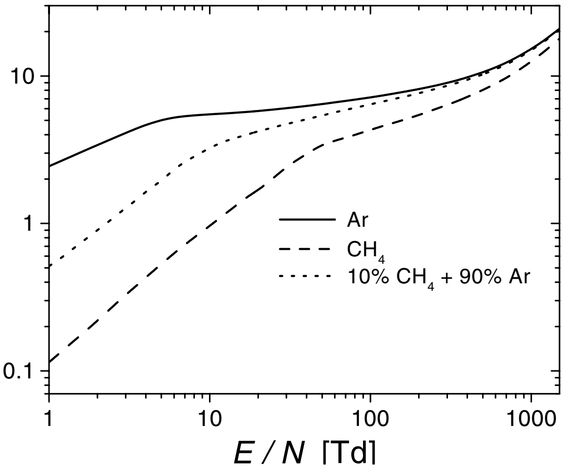

Fig. 3. E/N dependence of electron mean energies ε in Ar – solid line, CH_4 – dashed line and 10% $CH_4 + 90\%$ Ar mixture – dotted line.

a wide range for E/N, while its application for electrons was regarded as hopeless. Even in the case of ions Blanc's law had to be corrected to take into account inadequacy of its basic form [64]. In a similar fashion Wieland approximation can be valid only in mixtures of gases that have very similar electron distribution functions at the same E/N [11].

Recently there has been a significant development in application of Blanc's law for electrons. It has been proposed by many authors that Blanc's law should be applied for a common mean energy rather than common E/N, but Chiflikyan was the first [67] to make systematic studies to prove that this approach is valid. Jovanović et al. [66] have proved this procedure by using transport theory and also they have shown that for high E/N even the basic form of Blanc's law appears to be valid.

We propose a similar approach for predicting α/N for the mixtures [68]. The idea is to use α/N data at the same mean energy instead the data at the same E/N values:

$$\left[\frac{\alpha}{N}\left(\frac{E}{N}\right)\right]_m = \sum_z x_z \left[\frac{\alpha}{N}\left(\frac{E'}{N}\right)\right]_z, \tag{5}$$

where $(\alpha/N)_m$, E/N, $(\alpha/N)_z$ and $(E'/N)_z$ are taken at the same mean electron energy. We will label this approach as a "common mean energy" approach.

In order to test the common mean energy mixture law given by equation (5), we investigated mixtures of argon and methane for a wide range of abundances. The mixture of methane and argon was selected both because it is often used in particle detectors [69] and since addition of methane has a major effect on the mean electron energy as argon has no inelastic losses below 11 eV and methane has large vibrational excitations leading to dramatic changes in transport coefficients [70]. In other words the example chosen here is close to being the most difficult test case and most other cases would show much better results. We have used the ELENDIF code [71], where cross-sections for Ar were taken from [72], and for CH_4 from [73–77].

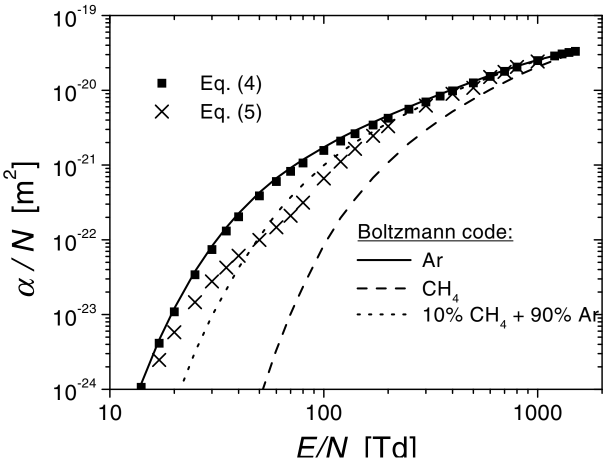

Fig. 4. Calculated electron ionization coefficients vs. E/N for Ar, CH_4 and for 10% $CH_4 + 90\%$ Ar mixture. The lines show results calculated by the Boltzmann code [71], while the symbols show data for the given mixture calculated by Wieland's approximation (Eq. (4)) – squares; and by common mean energy approximation (Eq. (5)) – crosses.

In the present analysis, we have employed a two term approximation to create a set of data for pure gases and mixtures in order to test the procedure. The procedure is not limited by the adequacy of the two term procedure. At the same time this should by no means be taken as a claim that a two term approximation is accurate and sufficient to model swarm studies [78] and discharges under all conditions. However, at moderate and high E/N the two term theory becomes sufficiently accurate for modelling of the mixture that was the object of this study as the two term code mainly fails in the region dominated by vibrational energy losses.

Figures 4 and 5 show $\alpha/N(E/N)$ for pure argon, pure methane and two different mixtures of these gases. Results were obtained by the Boltzmann code, along with the results obtained from mixture laws given by equations (4) and (5). Wieland approximation predicts the ionization coefficient that is almost exactly equal to abundance of Ar times the ionization coefficient in pure argon. In other words it does not take into account dramatic change in the mean energy (EEDF) due to addition of methane.

The common mean energy (CME) procedure is close to Wieland approximation at lower abundance and low E/N. This is surprising as in the mixture the mean energy is much lower than that for the pure argon, so majority of ionization comes from ionization of the methane rather than argon. Above few tens of Td the predicted value is close to the expected value for the mixture. Above 200 Td for 10% and above 300 Td for 30% of methane, Wieland approximation agrees well with both the expected and CME results.

Clearly, equation (4) cannot predict electron ionization in mixtures for all E/N. This is the result of the fact that the electron energy distribution is considerably different in mixtures and in individual gases at the same values of E/N. On the other hand, equation (5) provides

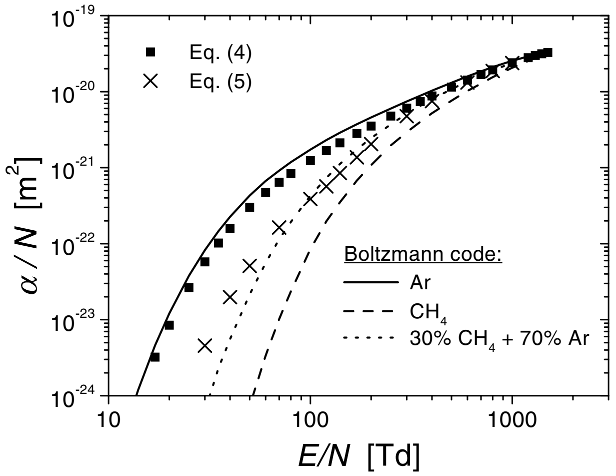

Fig. 5. Calculated ionization coefficients vs. E/N for Ar, CH_4 and 30% CH_4 + 70% Ar mixture. The lines show the results calculated by Boltzmann code [71]. Squares represent the data for the given mixture calculated by Wieland's approximation (Eq. (4)) and crosses represent data calculated by proposed common mean energy approach (Eq. (5)).

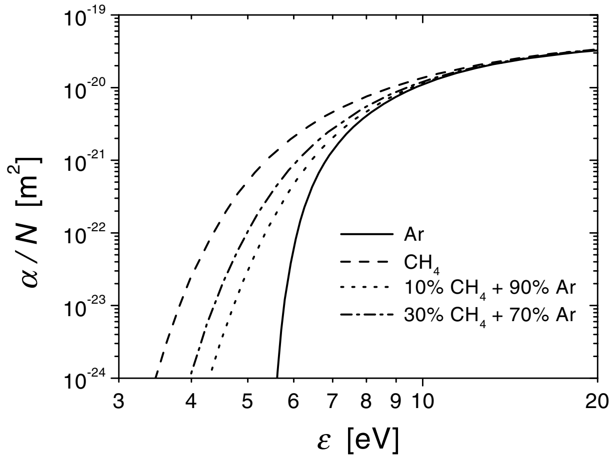

Fig. 6. Comparison of calculated ionization coefficients α/N as a function of mean electron energy ε. The solid line shows $\alpha/N(E/N)$ for pure Ar; dashed line for pure CH_4; dotted line for 10% CH_4 + 90% Ar mixture and dash-dot line for 30% CH_4 + 70% Ar mixture.

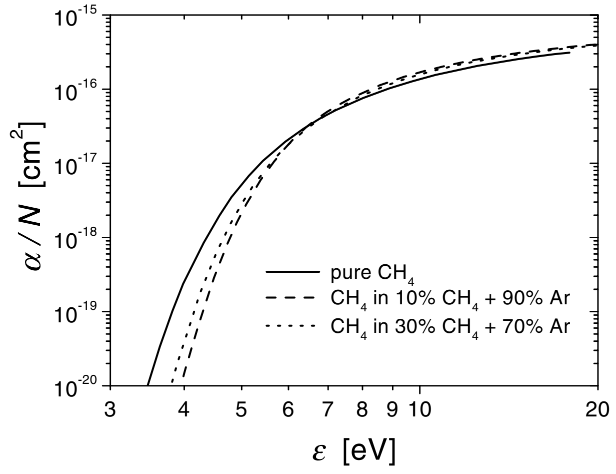

Fig. 7. Comparison of calculated ionization coefficients vs. mean electron energy for pure methane (solid line) and for methane alone in 10% CH_4 + 90% Ar and 30% CH_4 + 70% Ar mixtures (dashed and dotted line respectively). Boltzmann code used here enables us to calculate α/N for constituent gases in the mixture separately, and then total α/N is calculated as linear combination weighted by abundances. Here the CH_4 contribution is not normalized by the CH_4 abundance, so the differences represent the changes in EEDF.

a much better agreement with actual $(\alpha/N)_m$, especially in the mixtures with larger percentage of methane. Discrepancies that occur at the lower E/N values can be explained by results presented in Figure 6 where we show total ionization coefficients for pure gases and mixtures and in Figure 7 where we show only the effective coefficient for methane in pure methane and in the mixtures. If we consider α/N values at the same energy, it is obvious that for the lower mean electron energies ionization is quite different in the mixture and in the individual gases due to different electron energy distributions. In addition, in this range of energies the ionization of methane by far dominates over the ionization of argon. By increasing the percentage of methane in the mixture, the ionization coefficient $\alpha/N(\varepsilon)$ in the mixture becomes almost equal to that in pure methane (see Fig. 7). One may conclude however, that for the small E/N where in one gas (molecular in this case) the ionization is negligible and for the other (atomic) gas the ionization is considerable, the mixture law is bound to fail since ionization coefficients in two pure gases are different by many orders of magnitude. The procedure is expected to work reasonably well when the two ionization coefficients do not differ by more than two orders of magnitude.

The application of the technique for all other gases where we tried the same procedure (He–Xe and CH_4–N_2) gave much better results. Agreement was good even towards smaller E/N and also deviations are typically less than 3% at moderate and high E/N. In case of the mixture of molecular gases, the differences between ionization rates for the two constituent gases are smaller to begin with so the prediction of the mixture law do not appear as impressive and in that case even the Wieland approximation works well. In case of the He–Xe mixture, the differences between the two ionization coefficients are almost as large as for the Ar–CH_4 mixture. The agreement is much improved but there is a significant range at low E/N where one gas has no ionization while the other has a significant ionization and in that region the mixture technique begins to fail. The Ar–CH_4 mixture is the worst case of all that we tried (and it is unlikely that we could find a considerably more difficult case in general) but while it gives apparently the largest discrepancies, it also has to correct for the greatest differences.

In principle, the limitation of the procedure is in using the properties of the electron swarm that are due to the entire distribution function to describe processes that are determined only by the high energy tail of the electron distribution function. The procedure fails most seriously for low E/N where there is the largest difference between the mean electron energy and the threshold for ionization. Possible improvements could involve an application of the Momentum Transfer Theory (MTT) [79] that has developed techniques to correct the high energy properties in representation that is mainly associated with the mean energy. Robson has [80] employed exponential energy correction to the shape of the cross-section in order to make this correction. It has also been proposed [81] that the mean energy dependence of the rate coefficients may be used to replace the cross-sections in MTT, but that would require a more extensive knowledge of excitation coefficients in both pure gases and mixtures.

One could argue that if we need additional information on the mean energy the simplicity of the Wieland approximation may be lost and that in that case one may use Boltzmann calculations or Monte Carlo simulations. However, such calculations cannot be made easily for all the conditions and if one produces a lookup table of mean energies, ionization coefficients and E/N values application of CME procedure could be quite simple. Mean energies may be calculated on the basis of the cross-sections in the moderate energy range that need not be extremely accurate and in most fluid and hybrid plasma models the transport data are used by projecting through the mean energy rather than E/N. At the same time calculation of the ionization coefficient may depend critically on the availability of the data for electronic excitation and dissociation even when ionization cross-section is known exactly. In other words processes with losses at around 10 eV affect greatly the slope of the electron distribution function and its overlap with the ionization cross-section. Data for electronic excitation are often incomplete and data for dissociation into ground state fragments are not available for most gases.

In this paper we use plots of ionization rates as a function of E/N. This is a matter of tradition as the experimental data can be only obtained in this form. The advantage of the common mean energy procedure for mixtures which was so clearly shown for drift velocities and to a large degree for ionization rates favours plots as a function of mean energy in case when mixture data are to be determined. While using such plots would be an option, we would have to rely on a calculation of the mean energy. At the same time the E/N plots are purely based on experimental observables. Using mean energy to tabulate transport coefficients has become common in plasma modelling and the common mean energy procedure is directly compatible. At the same time we have to point out that the proposed procedure requires in any case calculation of the mean energy based on some available cross-section data. It may be relatively easy to produce mean energies which are mainly affected by vibrational excitation. On the other hand, ionization coefficients depend critically on all excitation processes including dissociation to neutral radicals which are often missing. Thus using the present procedure with experimental ionization rates and reasonable cross-section data may be the best option for gas mixtures.

One may argue that application of characteristics energies instead of mean energies would be practical and could lead to application of only experimental data. We have tested this possibility and it did not yield satisfactory results partly through anomalous relationship of eD_T/μ and mean energy for argon. In any case, to make this approach practical one would need a technique to estimate characteristic energies of mixtures, which is not available at the moment.

4 Summary

In this paper we have prepared an extensive list of parameters for using extended Townsend formula to provide analytic representation of the ionization coefficients. Application of this formula makes it possible to fit experimental ionization data in the entire range of the available data including the domain where the ionization rate decreases. Rather than being the source of accurate data the table and examples should rather been used as a warning against indiscriminate use of the basic Townsend formula and some older sources of its fitting parameters. While those sources quoted the range of applicability of the fitting parameters, those were sometimes used in the literature beyond those limits and certainly their applications could not be often justified having in mind more recent and more accurate ionization coefficients available in the literature. Our suggestion is to always extend the data base used here with data that are arguably of a better accuracy and perform fitting in as broad range of E/N as possible. One should bear in mind in particular such applications as volt-ampere characteristics of the Townsend discharges and evaluation of the secondary yields that may critically depend on details of α/N dependence on E/N [21].

Ionization coefficients are often used to model mixtures. Usual approximation to combine ionization coefficients weighted by the abundance (Wieland approximation) clearly fails in mixtures where the mean energies for the constituent gases at the same E/N are quite different. We have proposed a common mean energy technique to predict ionization coefficients for the mixture and it was shown to give a good prediction of ionization coefficient at moderate and high E/N. Of course, its application is nowhere near as successful as in the case of drift velocities. In particular it is not very useful close to the threshold where ionization in one gas is negligible while in the other it is considerable so the differences between the two pure gases and the mixture are several orders of magnitude. Procedure may be of use for analysis of data from particle detectors and understanding of synergism in gas breakdown in mixtures. The common characteristic energy technique is also tested, but for the mixture studied in this paper that involves argon, it does not give satisfactory results.

One should bear in mind that the example chosen here was the worst case for the mixtures that could be made on the basis of the data presented in Table 2. It was chosen to show limitations rather than the best case, and in most other examples, the procedure works very well, including the common characteristic energy procedure.

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